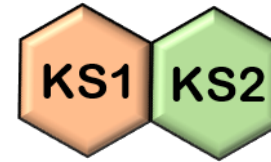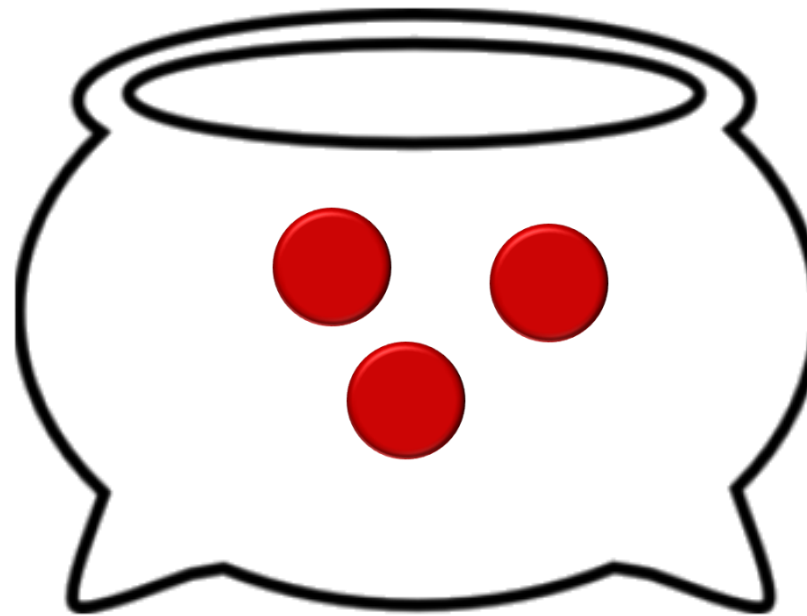# Nim Analysis

We looked at a version of simple nim in last week's challenge. Nim is an example of a combinatorial game. These are games that involve 2 people, have no chance elements and where there can't be a draw.

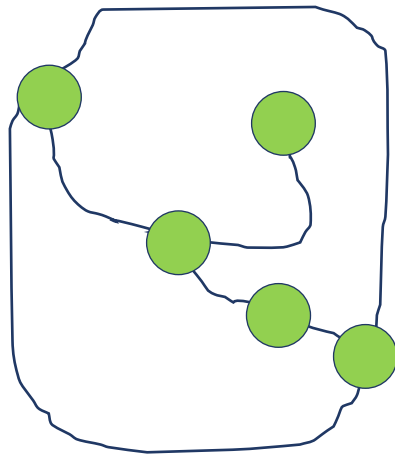| Number of counters	Best move
1	Take 1
2	Take 2
3	
4	
5	
6	
7	
8	
9	
10	

The table shows the two simplest examples of a game of nim and the best move.

Can you work out the best move if the game starts with 3, 4 or more counters? Should you take 1, 2 or let the other player start?

In simple nim you have one pot of counters, and players take turns removing 1 or 2 counters. The loser is the first person who cannot make a move. This is a common way to describe the end of combinatorial games.

Sprouts

Sprouts is another example of a combinatorial game.
In sprouts, you start with a number of dots on the page (these are the sprouts). Players take turns drawing a line to connect 2 sprouts or to connect a sprout to itself.
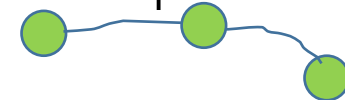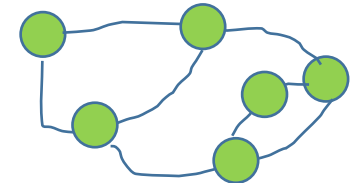
Example

Start with 2 sprouts

Player 1 draws a line connecting them and puts a new sprout on it.

Players continue like this until there are no sprouts with free spaces in the network.

This network has 2 sprouts with free spaces but they would need to cross a line to connect so the game is over.

Above is a game that has been started.
How many sprouts started the game?
How many moves have been played so far?
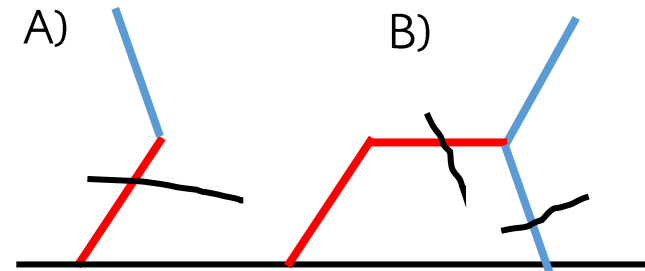Who do you think will win?

Hackenbush

Hackenbush is another combinatorial game.
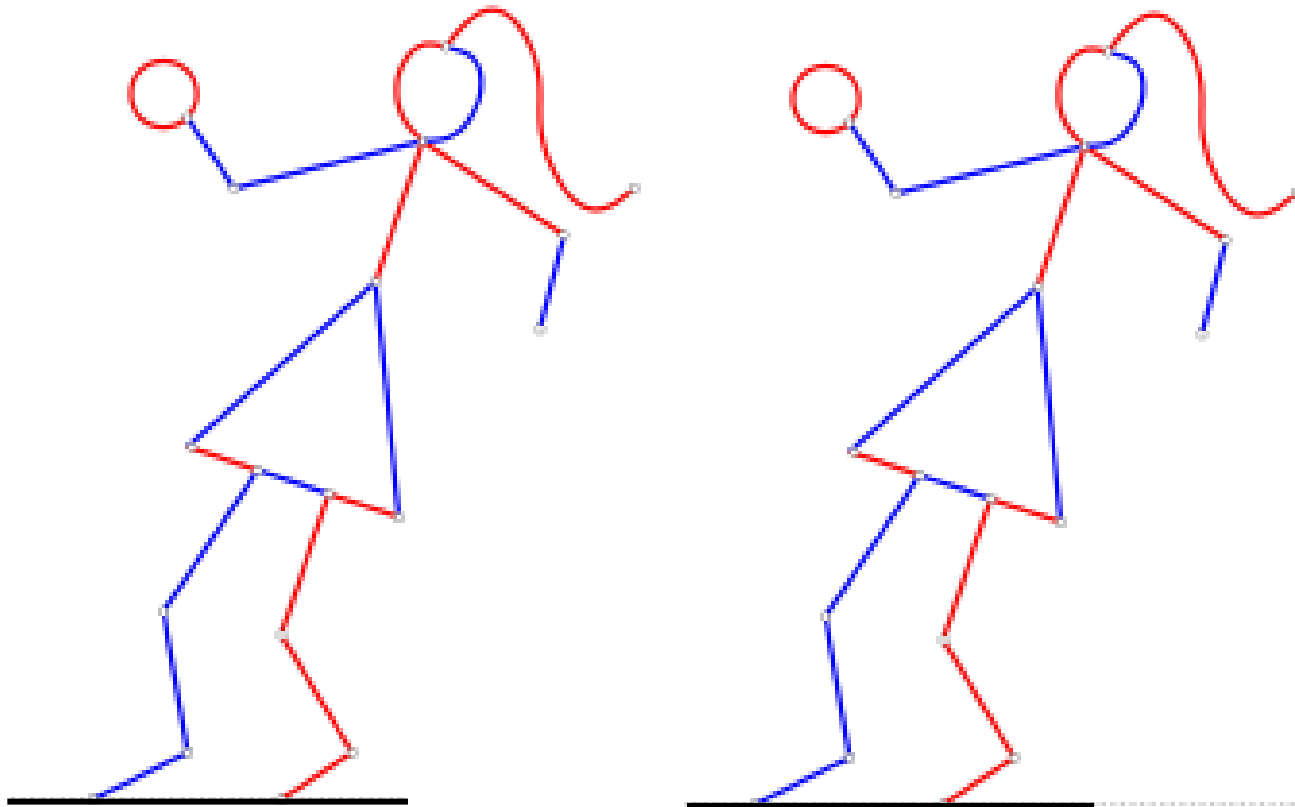
2 players choose a colour each. They then decide who starts and take turns crossing out a line of their colour. The last player to be able to make a legal move is the winner.

Other Rules

All lines must have a connection to the ground or they float off and can't be played

A) If red crosses out this line, the blue would float off and could not be played by blue. Red would win.
B) There are 2 paths that connect blue to the ground. If both are cut, the blue line at the top floats off.

Try playing the example on the left. Can you win if you go first?

Hackenbush Analysis

Which colour has the advantage in these games?

First player

Who wins?

Non Zero Game ✓ ✓

First player

Who wins?

First player

Who wins?

First player

Who wins?

First player

Who wins?

First player

Who wins?

Which of these games are the fairest games? i.e. the second player is most likely to win whichever colour they are

Zero Games

Games which are most likely to be lost by the player who goes first, no matter the colour, are called zero games.

First player

Who wins?

✓

✓

In these games, the second player is most likely to win. The first player can win but it will be due to mistakes or a poor choice of moves by the second player.
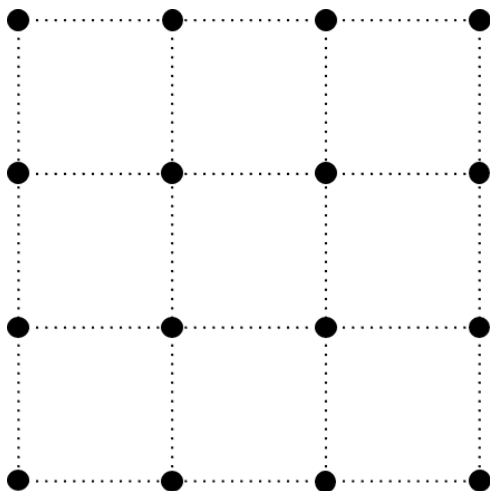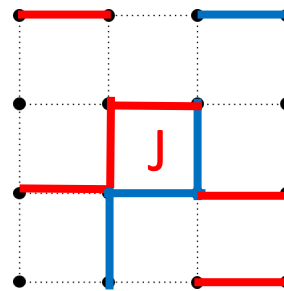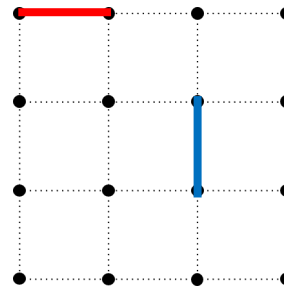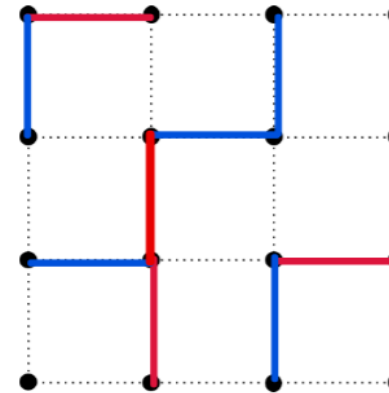
Dots and Boxes

KS1 KS2

Maths Explorers

Dots and boxes is a really well known combinatorial game. Players take turns to draw lines connecting 2 adjacent boxes. Lines can be vertical or horizontal.

When a player completes all 4 lines of a square, they put their initial in it and take another go.

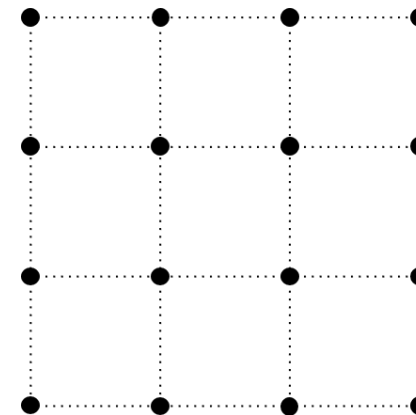The winner is the person who gets the most boxes.
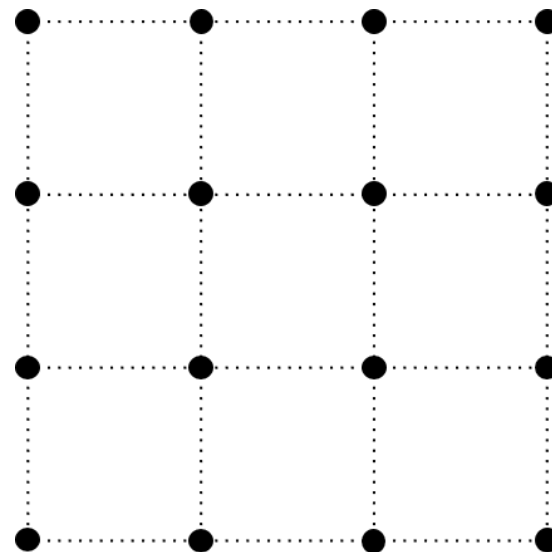
Analysis

Can you see the chains of boxes that will be collected?

Each of the 9 boxes here has 2 lines drawn and 2 yet to be drawn. Can you come up with another example of a game state like this?

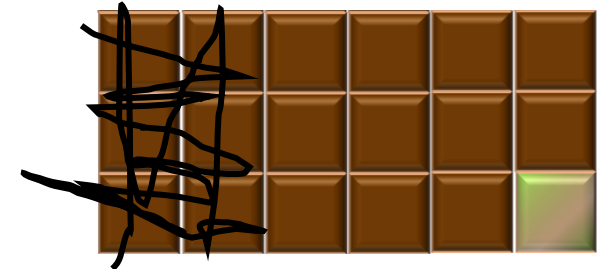Do you have any chains? How many? How big are they?

Yucky Choccy

Yucky Choccy is actually a version of Nim!
2 players take turns to break off a piece of chocolate. They can break of a number of rows or a number of columns of chocolate on each turn. Whichever player ends up with the yucky Choccy square is the loser.
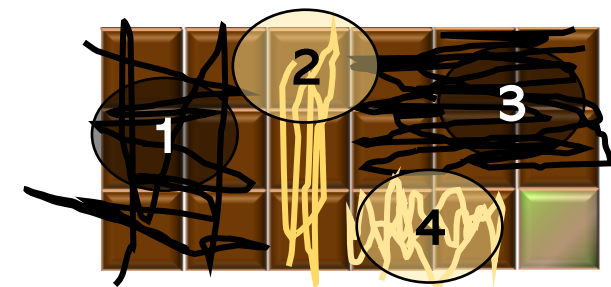Can you find any tactics to help you win?

How to Play

On your turn, cross out any number of whole rows or whole columns. You can choose either.

The game goes on until one player ends up with the yucky choccy square. In this game, black loses.